Influence of Radiative Absorption on the Establishment of Local Thermodynamic Equilibrium

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(Z. Naturforschg. **24 a**, 1492—1496 [1969]; received 24 May 1969)

By solving the coupled system of collisional-radiative rate equations for a homogeneous and steady-state plasma as a function of the radiative excitation rates one obtains the population densities of the ground and of the excited levels for any given degree of reabsorption. One finds that in hydrogen plasmas which are completely optically opaque towards all Lyman lines and partially optically opaque towards H_α the equilibrium populations will not be established for electron densities below 1×10^{16} cm^{-3}.

It is a well-known fact that population densities of the ground and of the excited states show departures from the Boltzmann and Saha values when the ionized gaz considered is optically thin and the electron density low. To obtain the equilibrium populations in an optically thin plasma one needs high electron densities. The latter ones must be so high that the plasma is in *all* transitions collision-dominated.

When a (homogeneous) plasma of constant temperature is completely opaque towards *all* radiative transitions the equilibrium populations are established for all levels (the ground level included) without any condition to be fulfilled by the electron density, since Planck's law holds for all frequencies. In this case we have a so-called radiation-dominated plasma.

Generally speaking, actual plasmas are very often neither collision- nor radiation-dominated, they lie mostly between these two extreme cases. Thus, there is a certain probability to find the plasma to be out of equilibrium. One often argues, however, that even in plasmas which are not collision-dominated the equilibrium populations belonging to a local temperature T_e are established due to sufficiently strong resonance absorption, since the first members of the resonance lines are generally completely opaque, although this may not be true for radiative transitions between the higher excited states.

We show in this paper that very often radiative absorption in the resonance lines can not compensate a lack of electronic collisions. To establish local thermodynamic equilibrium with respect to the ground state (so-called complete local thermodynamic equilibrium) one needs either high electron densities or — if this is not the case — substantial radiative absorption by the ground *and* by the excited states.

The numerical values given here are in a certain manner complementary to those published by different authors for optically thin [1–4] and partially optically thick plasmas [5,6].

Rate Equations

We consider the ideal case of homogeneous and stationary plasmas. All particles may have the same local temperature T. We replace the space-dependent emission and absorption of radiation governed by the radiative transfer equations by an *effective* absorption factor $1 - \Lambda$. For a transition belonging to any two levels i and j (with $i < j$), the coefficient Λ may be denoted by Λ_{ji}. For continuous absorption belonging to the level i, this factor be Λ_i. The introduction of these coefficients permits an idealized and very simple representation of the rates for photo-excitation $i \to j$ and photo-ionization $i \to c$:

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1 R. G. Giovanelli, Austral. J. Phys. **1**, 275 [1948].
2 R. W. P. McWhirter and A. G. Hearn, Proc. Phys. Soc. London **82**, 641 [1963].
3 H. W. Drawin, Ann Phys. Leipzig **16**, 195 [1965].
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5 A. G. Hearn, Proc. Phys. Soc. London **88**, 171 (1966).
6 H. W. Drawin, Excitation and percentage ionization of H, He I, and He II in a non-thermal steady-state plasma with variable resonance absorption, Report EUR-CEA-FC-302 [1965].

Denoting by $n_j A_{ji}$ the radiative de-excitation rate for a transition $j \to i$, the excitation rate due to the absorption of photons of frequency ν_{ij} minus the radiation induced emission is then given by $n_i(1 - \Lambda_{ji})A_{ji}$. By putting $\Lambda_{ji} = 1$ one has a plasma which is optically thin towards the transition $j \to i$. The value $\Lambda_{ji} = 0$ makes the plasma completely optically opaque towards this transition. By taking any value between zero and unity one may simulate any degree of reabsorption between these bound levels.

The radiative recombination rate for transitions $c \to i$ leading to a population of level i is given by $n_+ n_e R_i$, where R_i is the rate coefficient for spontaneous recombination. The effective photo-ionization rate minus the radiation induced recombination is then given by $n_+ n_e(1 - \Lambda_i) R_i$. By putting $\Lambda_i = 1$ one has no photo-ionization, and by putting $\Lambda_i = 0$ the boundfree transition $i \to c$ is complete optically opaque, or more generally speaking, the radiative recombination rate is completely balanced by an equivalent rate due to photo-ionization.

The relation between the coefficients Λ and the quantities in the radiative transport equation have been given in a special report[6].

The above given relations for the radiative transitions have been introduced in the coupled system of rate equations describing the level populations of the hydrogen atom, hydrogen-like ions, and the helium atom. The system contained all possible radiative and collisional transitions.

The total change of the number density n_i is governed by the following equation

$$\frac{\mathrm{D} n_i}{\mathrm{D} t} = \frac{\partial n_i}{\partial t} + \nabla_r \cdot (n_i \langle \boldsymbol{v}_i \rangle) = \left(\frac{\partial n_i}{\partial t}\right)_{\substack{\text{collision}\\ \text{radiation}}} \qquad (1)$$

where $\langle \boldsymbol{v}_i \rangle$ is the average drift velocity of particles in the level i. The right-hand side of Eq. (1) is the collision integral due to collisional-radiative processes.

It is composed of three parts which account for:

1. elastic and inelastic collisions with electrons,
2. elastic and inelastic collisions with heavy particles,
3. radiative processes.

Since we are only interested in the influence of the (inelastic) electronic collisions and the radiative processes upon the population densities, the contribution 2. may be dropped. Further, all elastic electronic collisions can be neglected.

For hydrogen and hydrogen-like ions one obtains for each level an expression of the following kind

$$\begin{aligned}
\left(\frac{\partial n_i}{\partial t}\right)_{\substack{\text{collision}\\ \text{radiation}}} = & \sum_{h<i}^{i-1} n_h \left[n_e C_{hi}^{(e)} + \frac{n_i}{n_h} (1 - \Lambda_{ih}) A_{ih} \right] \\
& + \sum_{k>i}^{p} n_k [n_e F_{ki}^{(e)} + A_{ki}] + n_e n_+ [R_i + n_e Q_i^{(e)}] \\
& - n_i \sum_{h<i}^{i-1} [n_e F_{ih}^{(e)} + A_{ih}] - n_i \sum_{k>i}^{p} \Bigg[n_e C_{ki}^{(e)} \\
& \qquad + \frac{n_k}{n_i} (1 - \Lambda_{ki}) A_{ki} \Bigg] \\
& - n_i \left[n_e S_i^{(e)} + \frac{n_+ n_e}{n_i} (1 - \Lambda_i) R_i \right] \qquad (1)
\end{aligned}$$

where $C_{hi}^{(e)}$ is the rate coefficient for electronic excitation $h \to i$ (with $h < i$), $F_{ki}^{(e)}$ that for electronic de-excitation $k \to i$, $n_e Q_i^{(e)}$ the rate coefficient for three-body collisional recombination, and $S_i^{(e)}$ that for ionization due to electronic collisions. The n_i, n_k, etc. denote the population densities of the levels with principal quantum numbers i, k, etc. The highest still bound level has the quantum number p approximately given by

$$p \cong Z^2 \langle d_+ \rangle / a_0 = 1.08 \times 10^4 Z n_+^{-1/6}. \qquad (2)$$

The quantity $\langle d_+ \rangle$ is the mean distance between the charged particles, a_0 the first Bohr radius. Our system contained 25 coupled equations. When Eq. (2) gave values $p > 25$ we solved the system containing 25 equations. The cross sections used for the evaluation of the system (1) may be found in a special report[7]. The rate coefficients have been calculated for a Maxwellian velocity distribution of the electrons.

For a homogeneous and steady-state plasma holds

$$\nabla_r \cdot (n_i \langle \boldsymbol{v}_i \rangle) \equiv 0; \quad \frac{\partial n_i}{\partial t} \equiv 0; \quad i = 1, 2, \ldots, p. \qquad (3)$$

and one obtains for the collision integrals (1) the condition

$$\left(\frac{\partial n_i}{\partial t}\right)_{\substack{\text{collision}\\ \text{radiation}}} = 0, \qquad i = 1, 2, \ldots, p. \qquad (4)$$

Results

By putting numerical values for n_e, T_e, Λ_i and Λ_{ij} into the system (4) the population densities $n_1, n_2, \ldots, n_p$, can be calculated.

[7] H. W. Drawin, Collision and transport cross sections, Report EUR-CEA-FC-383, March 1966 (revised January 1967).

To see wether the solutions n_i deviate from the equilibrium populations $n_i^{(\mathrm{Saha})}$ or not one may put the solutions for n_i into the following form (see refs. [8, 9]):

$$n_i = b_i \, n_i^{(\mathrm{Saha})} . \tag{3}$$

When b_i is equal to unity the level i shows the equilibrium population given by the Saha-Eggert equation in the following form

$$\frac{n_+ n_\mathrm{e}}{n_i} = \frac{2\,g_+}{g_i} \frac{(2\,\pi\,m_\mathrm{e}\,k\,T_\mathrm{e})^{3/2}}{h^3} \exp\left\{-\frac{E_i - \Delta E_{z-1}}{k\,T_\mathrm{e}}\right\} , \tag{4}$$

where n_+ is the number density of the ground state particles of the next higher ionization stage and ΔE_{z-1} the lowering of the ionization energy. It is well-known that b_i approaches always unity for sufficiently excited levels, see e.g. refs. [1–4].

For quantitative spectroscopy one needs not only the b_i-values. Of special interest is the knowledge of the population density n_i of an excited level $i > 1$ relative to that of the ground level $i = 1$. In the case of local thermodynamic equilibrium the ratio $n_i^{(\mathrm{Saha})}/n_1^{(\mathrm{Saha})}$ is simply given by the Boltzmann ratio $(g_i/g_1) \exp[-(E_1 - E_i)/k T_\mathrm{e}]$. For a non-equilibrium plasma, however, the actual ratio n_i/n_1 may be smaller than unity or larger.

The departure from the Boltzmann ratio shall now be characterized by the quantity a_i defined by

$$\frac{n_i}{n_1} = a_i \, (n_i^{(\mathrm{Saha})}/n_1^{(\mathrm{Saha})}) . \tag{5}$$

In the case of local thermodynamic equilibrium holds $a_i = 1$, for a steady-state non-L.T.E. plasma holds $a_i < 1$. The values b_i and a_i have been calculated for hydrogen, hydrogen-like ions and for helium. We only present the results for hydrogen.

In the Figs. 1 and 2 we show the calculated coefficients b_1 for different degrees of reabsorption, beginning with the optical thin plasma (all $\Lambda_i = \Lambda_{ij} = 1$, see curve 1). The curves 2 to 8 are for increasing absorption in the different spectral series. One sees that in the completely optical thin case the equilibrium population ($b_1 = 1$) can only be obtained for electron densities larger than $10^{18}\ \mathrm{cm}^{-3}$. When all resonance lines are optically opaque ($\Lambda_{j1} = 0$) the Saha-population is established

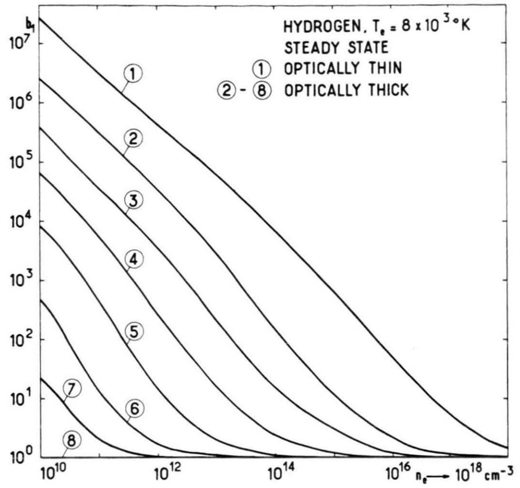

Fig. 1. The coefficient b_1 (ground level) for the electron temperature $T_\mathrm{e} = 8 \times 10^3$ °K. *Curve* **1**: all transitions optically thin ($\Lambda_{ji} = 1$, $\Lambda_i = 1$); *Curve* **2**: Ly-α optically opaque, all other transitions optically thin ($\Lambda_{21} = 0$); *Curve* **3**: All Lyman lines optically opaque ($\Lambda_{j1} = 0$); *Curve* **4**: All Lyman lines, the Lyman continuum, and the H_α line are optically opaque ($\Lambda_{j1} = \Lambda_1 = \Lambda_{32} = 0$); *Curve* **5**: All Lyman and Balmer lines, and the Lyman and Balmer continua are optically opaque ($\Lambda_{i1} = \Lambda_1 = \Lambda_{j2} = \Lambda_2 = 0$); *Curve* **6**: All Lyman, Balmer, Paschen lines and the Lyman, Balmer and Paschen continua are optically opaque; *Curve* **7**: All Lyman, Balmer, Paschen and Pfund lines, and the Lyman, Balmer, Paschen, and Pfund continua are optically opaque; *Curve* **8**: Complete radiative balancing in all transitions (25 levels).

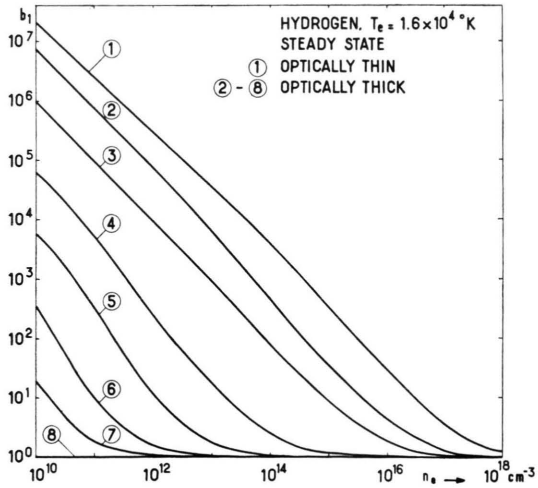

Fig. 2. The coefficient b_1 for the electron temperature $T_\mathrm{e} = 1.6 \times 10^4$ °K. The meaning of the curves is as in Fig. 1.

for $n_\mathrm{e} \geqq 3 \times 10^{16}\ \mathrm{cm}^{-3}$. To get $b_1 = 1$ for electron densities $n_\mathrm{e} < 3 \times 10^{16}\ \mathrm{cm}^{-3}$ one needs substantial reabsorption in the higher spectral series.

[8] D. H. MENZEL, Astrophys. J. **85**, 330 [1937].
[9] D. H. MENZEL and J. G. BAKER, Astrophys. J. **86**, 70 [1937].

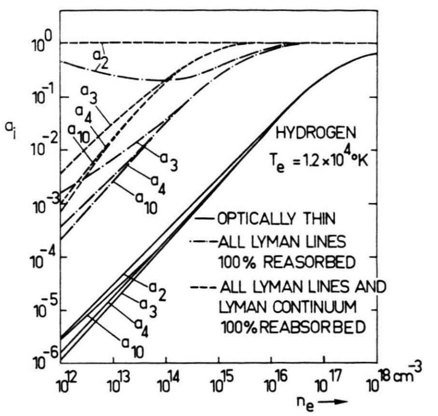

Fig. 3. The coefficients a_i for the electron temperature $T_e = 1.2 \times 10^4$°K and for three different cases: ——— (solid curves): all transitions optically thin ($\Lambda_i = \Lambda_{ji} = 1$); ·—·—: All Lyman lines optically opaque ($\Lambda_{j1} = 0$); ------: All Lyman lines and the Lyman continuum optically opaque ($\Lambda_{j1} = \Lambda_1 = 0$).

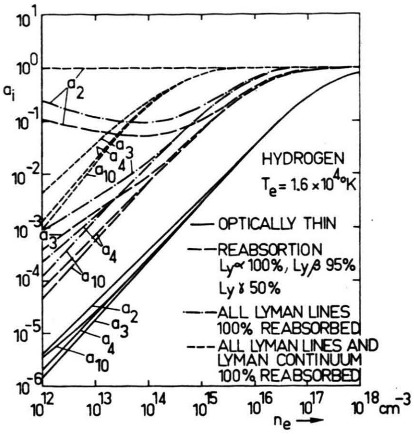

Fig. 4. The coefficients a_i for the electron temperature $T_e = 1.6 \times 10^4$°K and for four different cases: ——— (solid curves): All transitions optically thin ($\Lambda_{ji} = \Lambda_i = 1$); ——— Ly-$\alpha$: 100% reabsorbed ($\Lambda_{21} = 0$) Ly-$\beta$: 95% reabsorbed ($\Lambda_{31} = 5 \times 10^{-2}$), Ly-$\gamma$: 50% reabsorbed ($\Lambda_{41} = 0.5$) all other transitions optically thin; ·—·— All Lyman lines complete optically opaque ($\Lambda_{j1} = 0$); ------ All Lyman lines and the Lyman continuum optically opaque ($\Lambda_{j1} = \Lambda_1 = 0$).

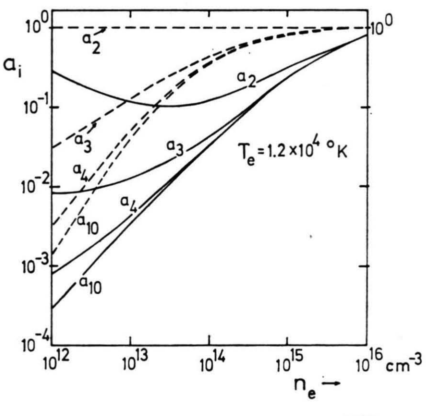

Fig. 5. The coefficients a_i for $T_e = 1.2 \times 10^4$°K. ——— (solid curves): All Lyman lines optically opaque ($\Lambda_{j1} = 0$) and 90% reabsorption in the H_α line ($\Lambda_{32} = 0.1$); ------ (broken curves): All Lyman lines and the Lyman continuum optically opaque ($\Lambda_{j1} = \Lambda_1 = 0$), and 90% reabsorption in the H_α line ($\Lambda_{32} = 0.1$).

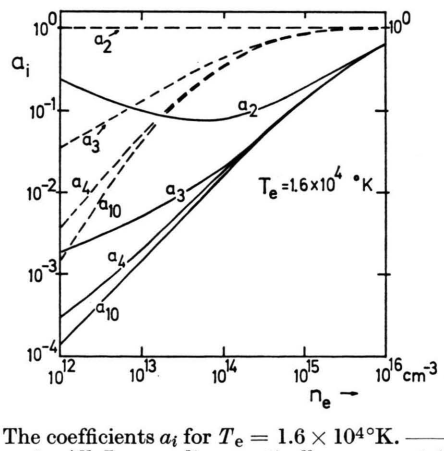

Fig. 6. The coefficients a_i for $T_e = 1.6 \times 10^4$°K. ——— (solid curves): All Lyman lines optically opaque ($\Lambda_{j1} = 0$) and 50% reabsorption in the H_α line ($\Lambda_{32} = 0.5$). ------ (broken curves): All Lyman lines and the Lyman continuum optically opaque ($\Lambda_{j1} = \Lambda_1 = 0$) and 50% reabsorption in the H_α line $\Lambda_{32} = 0.5$).

The coefficients a_i for the levels $i = 2, 3, 4$, and 10 are shown in the Figs. 3 to 6 for two different electron temperatures. These figures show clearly that resonance absorption can considerably extend the validity of complete local thermodynamic equilibrium to lower electron densities. However, even in the case of completely optically opaque Lyman series and a partially optically thick H_α-line there always occur deviations from complete L.T.E. for electron densities below 1×10^{16} cm^{-3}.

For all ionized particles the situation is much more severe than for hydrogen due to the fact that the spontaneous transition probabilities increase like $Z^4 A_{ji}$ whereas the collisional rate coefficients decrease like Z^{-3}.

The numerical values published in this paper refer to an idealized homogeneous plasma. Under actual conditions one has to account for the inhomogeneous character of all plasmas. Generally speaking, one has to calculate the population densities from a coupled system of rate equations coupled with the corresponding radiative rate equations. The numerical values given here may nevertheless give a very good idea on what happens when substantial radiative absorption exists.

The decrease of the plasma temperature towards the boundary will slightly favour the establishment of L.T.E. compared to the values given in the figures. This is due to the fact that the outer cold plasmas zones are under the influence of the radiation field originating from the central hot zone. On the other hand, the radiation field of the cold boundary zones will practically not influence the population densities of the central hot part of temperature T_e.

One concludes from these calculations that it will be very difficult to establish complete L. T. E. in the level system of hydrogen below electron densities of about 1×10^{16} cm^{-3}.

I thank Miss F. EMARD for having written the computer programme.

Bestimmung des Elektronendichteprofils in Plasmen mit Hilfe von Hohlraumresonatoren

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(Z. Naturforschg. 24 a, 1496—1501 [1969]; eingegangen am 4. Juli 1969)

Determination of Plasma Electron Density Profiles by Means of Resonant Cavities

The shift of the resonant frequency of microwave cavities excited in TM_{0m0} ($m=1, 2, 3$), TM_{l10} ($l=1, 2$), TE_{01n} ($n>0$, integer) modes under the influence of a plasma is calculated for arbitrary radii of plasma and cavity for Bessel profile and common parabolic profiles.

In measuring the frequency shift for two of the cited cavity mode types an approximate electron density profile can be determined from the submitted diagrams.

Die Resonanzfrequenz ω_0 eines Mikrowellen-Hohlraumresonators verändert sich unter dem Einfluß eines Dielektrikums im Resonator. Wird ein Plasma als Dielektrikum verwendet, so ist es möglich, aus der Verschiebung $\Delta\omega$ der Resonanzfrequenz die Elektronendichte n des Plasmas zu bestimmen [1]. Bisher durchgeführte Rechnungen setzten ein bestimmtes Verhältnis der Radien der Plasmasäule und des Resonators und einschränkende Elektronendichteprofile voraus, oder bedingten weitere umfangreichere Rechnungen zur Bestimmung des Profils [2–8].

Für Resonatoren der Typen TM_{010}, TM_{020}, TM_{030}, TM_{110}, TM_{210}, TE_{01n} wird hier die Frequenzverschiebung $\Delta\omega$ und die Güteänderung $\Delta(1/Q)$ für beliebige Verhältnisse von Plasmaradius zu Resonatorradius bei beliebiger Resonanzfrequenz für Besselsche J_0-, rechteckförmige und allgemeine parabolische Dichteverteilungen berechnet. Aus Messungen in zwei Resonatormoden am selben Plasma läßt sich das Dichteprofil bestimmen.

1. Berechnung

Nach SLATER [9] beträgt die Frequenzverschiebung $\Delta\omega$ und die Güteänderung $\Delta(1/Q)$ eines Hohlraumresonators mit der ungestörten Resonanzfrequenz ω_0 und der ungestörten Mikrowellenfeldstärke $E(r)$ bei

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[2] S. C. BROWN, Proc. 2nd Intern. Conf. Peaceful Uses of Atomic Energy (Geneva) **32**, 394 [1958].
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